Report on the Trimester Program

K-Theory and Related Fields

May - August 2017

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Topics

Since its beginning in the celebrated work of Grothendieck on the Riemann-Roch theorem, K-theory has been a tool for understanding a wide-ranging family of mathematical structures and their invariants. Topological spaces, rings, algebraic varieties and operator algebras are the dominant examples. The invariants range from characteristic classes in cohomology, through determinants of matrices, to Chow groups of varieties, as well as to traces and indices of elliptic operators. They have applications in a variety of subjects, including algebraic geometry, number theory, algebraic and geometric topology, representation theory, geometric and functional analysis, and mathematical physics. Thus K-theory is notable for its connections with other branches of mathematics. Because of this, mathematicians from many areas have become interested in the subject, and they all look at it from their own perspective. On the one hand, this is the richness and appeal of K-theory. On the other hand, it makes it hard to see a global perspective.

The Trimester Program on K-Theory and Related Fields was organized around these various subbranches and applications of the subject.

Goals

The primary goal of the Trimester Program was to advance the various branches of K-theory through interaction between those working on several aspects of the subject. Informal interactions took place at HIM throughout the course of the trimester, and regular seminars and working groups covered many different topics. In addition, there was a summer school for PhD students and young researchers to expose them to a variety of current areas of research.

Organization

Three different major workshops were held:

1. K-theory in algebraic geometry and number theory, May 15–19, 2017, with speakers Ben Antieau, Joseph Ayoub, Tom Bachmann, Christian Dahlhausen, Lie Fu, Jens Hornbostel, Annette Huber-Klawitter, Ryomei Iwasa, Satoshi Kondo, Amalendu Krishna, Matthew Morrow, Thomas Nikolaus, Paul-Arne Østvær, Oliver Röndigs, Shuji Saito, Marco Schlichting, Peter Scholze, and Gonçalo Tabuada;

2. K-theory and related fields, June 26–30, 2017, with speakers Federico Binda, Emanuele Dotto, Shane Kelly, Moritz Kerz, Guido Kings, Cary Malkiewich, Michael Mandell, Justin Noel, Charanya Ravi, Birgit Richter, Iuliia Semikina, Vasudevan Srinivas, Wolfgang Steimle, Georg Tamme, Charles Weibel, Christoph Winges;

3. K-theory in topology and non commutative geometry, August 21–25, 2017, with speakers Francesca Arici, Sara Arklint, Christian Ausoni, Paulo Carrillo Rouse, Joachim Cuntz, Grigory Garkusha, Pierre Julg, Max Karoubi, Daniel Kasprowski, Ralph Kaufmann, Wolfgang Lück, Ralf Meyer, Kristian Moi, Ryszard Nest, Erik Pedersen, Birgit Richter, Michael Weiss, and Guoliang Yu.

In addition, a Summer School was held June 19–23, 2017, just before the second workshop, with courses by Arthur Bartels (Münster) on K-theory of group rings, by Moritz Kerz (Regensburg) on Algebraic K-theory and descent for blow-ups, by Marc Levine (Essen) on Refined enumerative geometry, by Andreas Thom (Dresden) on Algebraic vs. topological K-theory, and by Charles Weibel (Rutgers) on K-theory of algebraic varieties.

Besides these major events, there was a regular seminar that usually met at least once a week during non-workshop weeks. Talks included:

- Manh Toan Nguyen: Equivariant motivic cohomology;

- Vivek Sadhu: A relative version of Weibel's vanishing conjecture;

- Rufus Willett: Classification of C^*-algebras and computing K-theory;

- Lars Hesselholt: K-theory of division algebras over local fields;

- Alexander Berglund: A dg Lie algebra model for the block diffeomorphism group;

- Thomas Nikolaus: On topological cyclic homology I and II;

- Alexander Varchenko: Elliptic dynamical quantum group $E_{\tau,h}(\mathfrak{gl}_2)$ and elliptic equivariant cohomology of the cotangent bundles of Grassmannians;

- Cary Malkiewich: Periodic orbits and equivariant traces;

- Adeel Khan: Motivic infinite loop spaces;

- Shantanu Dave: Geometric hypoellipticity and topological invariants;

- Irakli Patchkoria: THH of differential graded algebras and exotic equivalences;

- Heng Xie: Algebraic KR-theory of algebraic varieties; and

- Bram Mesland: A Hecke module structure on the KK-theory of arithmetic groups.

There were also four study groups, on dg categories and non-commutative motives (organized by Patrick McFaddin and Lie Fu), on topological Hochschild and cyclic homology (organized by Alice Hedenlund and Calvin Woo), on the paper of Kerz, Saito, and Tamme "Towards a Non-Archimedean analog of the Bass-Quillen conjecture" (organized by Christian Dahlhausen), and on the Grayson-Suslin spectral sequence (organized by Heng Xie).

Results

The Trimester program resulted directly in 17 papers written (LINK) and 71 videos of recorded lectures, with at least 1,438 views on **YouTube**. Like the many formal and informal lectures, the papers worked on during the program reflect the breadth of mathematics making use of K-theoretic techniques. They include work in geometric representation theory (for example, preprint 2017b1); the theory of motives and algebraic cycles (for example 2017b10, 2017b14, 2017b05); the theory of non-commutative motives (2017b08, 2017b09); twisted K-theory of compact Lie groups (2017b06);

topological Hochschild homology (2017b02); applications of K-theory in physics (2017b12, 2017b13); algebraic cobordism (2017b16); and more. This is all, of course, in addition to the more intangible benefit of the interactions that took place over the course of the Trimester.